Problem 1.
(a) The points $(3, -1, 2)$ and $(-1, 3, -4)$ are the endpoints of a diameter of a sphere.
   (i) Determine the center and radius of the sphere.
   (ii) Find an equation for the sphere.

(b) Given the vectors $a = 2i - j + 2k, b = 3i + 2j - k, c = i + 2k$.
   (i) Calculate $2a \cdot (b - 3c)$.
   (ii) Determine the vector projection of $c$ onto $b$.
   (iii) Find the cosine of the angle between $a$ and $b$.
   (iv) Find a unit vector that is perpendicular to the plane determined by $a$ and $c$.

Problem 2. Given the planes $P_1 : 2(x - 1) - (y + 1) - 2(z - 2) = 0, P_2 : 4x - 2y + 5z = 3$, and the point $Q : (-2, 7, 4)$.
(a) Determine whether $P_1$ and $P_2$ are parallel, coincide, perpendicular, or none of the preceding.
(b) Find an equation for the plane through $Q$ which is parallel to $P_1$.
(c) Determine scalar parametric equations for the line through $Q$ which is parallel to the line of intersection of $P_1$ and $P_2$.

Problem 3. The position of an object at time $t$ is given by:
\[ r(t) = e^{-t}i + e^t j - t\sqrt{2}k, \quad 0 \leq t < \infty \]
(a) Determine the velocity and the speed of the object at time $t$.
(b) Determine the acceleration of the object at time $t$.
(c) Find the distance that the object travels during the time interval $0 \leq t < 1$.

Problem 4.
(a) A curve $C$ in the plane is defined by the parametric equations: $x = t^2 + 1, \ y = \frac{4}{3}t^3 - 1$.
   (i) Find the length of $C$ from $t = 0$ to $t = 2$.
   (ii) Find the curvature of $C$ at $t = 1$.

(b) The vector function $r(t) = \sin 2t i - \cos 2t j + t\sqrt{5}k$ determines a curve $C$ in space.
   (i) Find the unit tangent vector and the principal unit normal.
   (ii) Determine the curvature of $C$ at time $t$.
   (iii) Determine the tangential and normal component of the acceleration vector.
Problem 5. Let \( f(x,y) = x \ln(x/y) + xy^2 \).
(a) Calculate \( f_{xx} \) and \( f_{yz} \).
(b) Determine the directional derivative of \( f \) at the point \((2,2)\) in the direction of the vector \( a = i - 2j \).
(c) Suppose that \( x = st e^t \) and \( y = 2se^t \). Calculate \( \frac{\partial f}{\partial t} \).
(d) Determine an equation for the tangent plane to the surface \( z = f(x,y) \) at the point \((2,2,8)\) on the surface.

Problem 6. Let \( f(x,y,z) = 2xy^2 + 2yz^2 + 2x^2z \).
(a) Determine the maximum directional derivative of \( f \) at the point \((1,-1,1)\).
(b) Find the directional derivative of \( f \) at the point \((-2,1,-1)\) in the direction parallel to the line \( x = 34t, y = 2 - t, z = 3t \).
(c) Determine symmetric equations for the normal line to the level surface \( f(x,y,z) = -2 \) at the points \((-1,2,1)\).
(d) Suppose the \( x = t^2 + 1, y = 2t, z = t^3 \). Calculate \( \frac{df}{dt} \).

Problem 7.
(a) Let \( f(x,y) = 3x^2y - 2y^2 - 3x^2 - 8y + 2 \).
(i) Find the stationary points of \( f \).
(ii) For each stationary point \( P \) found in (i), determine whether \( f \) has a local maximum, a local minimum, or a saddle point at \( P \).
(b) Determine the minimum value of \( f(x,y) = 2x^2 + xy - y^2 + 1 \) subject to the constraint \( 2x + 3y = 16 \).

Problem 8.
(a) Find the absolute maximum and absolute minimum values of \( f(x,y) = x^2 + 2y^2 - 2x + 2 \) on the closed disk \( D : x^2 + y^2 \leq 4 \).
(b) Find the absolute maximum and absolute minimum values of \( f(x,y) = 2 + 2x + 2y - x^2 - y^2 \) on the closed triangular region bounded by the lines \( x = 0, y = 0, x + y = 9 \).
Problem 9.
(a) An open-topped rectangular container is to have a volume of 32 cubic meters. Find the dimensions of the container having the smallest surface area.

(b) The temperature $T$ at a point $(x, y, z)$ on the sphere $x^2 + y^2 + z^2 = 1$ is given by $T(x, y, z) = 400xyz^2$. What are the maximum and minimum temperatures?

Problem 10.
(a) Given the repeated integral $\int_0^2 \int_{x^2}^4 2x \cos(y^2) \, dy \, dx$. Determine an equivalent repeated integral with the order of integration reversed. Evaluate one of the two integrals.

(b) Express the area of the region bounded by the curves $y = 2x$ and $y = x^2$ by a repeated integral integrating: (i) first with respect to $y$, then with respect to $x$; (ii) first with respect to $x$, then with respect to $y$.

(c) Use a double integral to find the volume of the solid $S$ in the first octant that is bounded above by the surface $z = 4 - x^2 - y^2$, below by the $x, y$-plane, and on the sides by the planes $y = 0$ and $y = x$.

Problem 11.
(a) Evaluate $\int \int \int_T 2yz \, dx \, dy \, dz$ where $T$ is the solid in the first octant bounded above by the cylinder $z = 4 - x^2$ below by the $x, y$-plane, and on the sides by the planes $z = 0$, $x = 0$, $y = 2x$, and $y = 4$.

(b) Set up a triple integral in cylindrical coordinates that gives the volume of the solid in the first octant that is bounded above by the hemisphere $z = \sqrt{2 - x^2 - y^2}$, below by the paraboloid $z = x^2 + y^2$ and on the sides by the $x, z$- and $y, z$-planes.

(c) Set up a triple integral in spherical coordinates that gives the volume of the solid that lies outside the cone $z = \sqrt{x^2 + y^2}$ and inside the hemisphere $z = \sqrt{1 - x^2 - y^2}$.

Problem 12.
(a) Let $\mathbf{h}(x, y, z) = xy\mathbf{i} + y\mathbf{j} - yx\mathbf{k}$, and let $C$ be the curve given by $\mathbf{r}(u) = u\mathbf{i} + u^2\mathbf{j} + 2u\mathbf{k}$, $0 \leq u \leq 1$. Calculate $\int_C \mathbf{h}(\mathbf{r}) \cdot d\mathbf{r}$.

(b) Show that $\mathbf{h}(x, y) = (6xy - y^3)\mathbf{i} + (4y + 3x^2 - 3xy^2)\mathbf{j}$ is the gradient of a function $f$. Use this information to calculate $\int_C \mathbf{h}(\mathbf{r}) \cdot d\mathbf{r}$ where $C$ is the curve consisting of the line segment from $(0, 0)$ to $(2, 4)$ and the parabola $y = x^2$ from $(2, 4)$ to $(3, 9)$.
Problem 13.

(a) Let \( h(x, y) = 2xy^3 i + 4x^2y^2 j \). Calculate \( \oint_C \mathbf{h}(\mathbf{r}) \cdot d\mathbf{r} \) where \( C \) is the boundary of the triangular region in the first quadrant bounded by the x-axis, the line \( x = 1 \) and the curve \( y = x^3 \).

(b) Let \( g(x, y) = (2xy + e^x - 3) i + (x^2 - y^2 + \sin y) j \). Calculate \( \oint_C \mathbf{g}(\mathbf{r}) \cdot d\mathbf{r} \) where \( C \) is the ellipse \( 4x^2 + 9y^2 = 36 \).

(c) Use Green’s Theorem to find the area enclosed by the astroid \( \mathbf{r}(u) = \cos^3 u \mathbf{i} + \sin^3 u \mathbf{j}, \ 0 \leq u \leq 2\pi \).