EXERCISES FOR MATH 2331 DUE APRIL 15

(1) Find an orthogonal basis for the column space of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

(2) Find a least-squares solution of $A\mathbf{x} = \mathbf{b}$ by (a) constructing the normal equations for $\hat{\mathbf{x}}$ and (b) solving for $\hat{\mathbf{x}}$. Then (c) find the distance from $A\hat{\mathbf{x}}$ to $\mathbf{b}$.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

(3) Find the equation $y = ax + b$ of the least-squares line that best fits these data points:

$$(-2, 4)\ (-1, 2)\,,\ (1, 0)\,,\ (2, 0)\,.$$

(4) Let

$$\mathbf{v} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix},\ \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix},\ \mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix},\ \mathbf{u}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

Let $W = Span\,[\mathbf{u}_1,\ \mathbf{u}_2,\ \mathbf{u}_3]$. Express $\mathbf{v}$ as the sum of a vector in W and a vector orthogonal to W.