EXERCISES FOR MATH 2331 DUE APRIL 27

(1) Find a singular value decomposition for the rotation matrix $R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$.

(2) (a) Orthogonally diagonalize $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (find P and D).

(b) Find a spectral decomposition of A.

(c) For $\mathbf{x} \in \mathbb{R}^3$, let $Q(\mathbf{x}) = \mathbf{x}^T A\mathbf{x}$. Find the maximum and minimum values of Q on the unit sphere $S = \left[\mathbf{x} \in \mathbb{R}^3 : \mathbf{x}^T\mathbf{x} = 1\right]$

(3) For each of the quadratic forms Q_i below, determine if Q is positive definite, negative definite, or indefinite. Then find an orthogonal change of coordinates that transforms Q_i into a quadratic form with no cross-product terms.

(a) $Q_1(x_1,\ x_2) = -2x_1^2 + 2x_1x_2 + -2x_2^2$.

(b) $Q_2(x_1,\ x_2) = 4x_1^2 + 2x_1x_2 + 4x_2^2$.

(c) $Q_3(x_1,\ x_2) = -x_1^2 + 4x_1x_2 - x_2^2$.