Math 2331
January 23, 2020 Homework 2 Name ____________________

1. Compute the product $A\mathbf{x}$, where

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 7 & -8 & 9 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

2. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \ \mathbf{b}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \ \mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

   (a) Determine whether $\mathbf{b}_1$ is in $Span\,\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$.

   (b) Determine whether $\mathbf{b}_2$ is in $Span\,\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$.

   (c) If $\mathbf{b}_1$ or $\mathbf{b}_2$ is in $Span\,\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$, find weights $x_1,\ x_2,\ x_3$ such that $\mathbf{b}_i = x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3$.

3. Let

$$A = \begin{bmatrix} 1 & -2 & 4 \\ 1 & -1 & 3 \\ 1 & 3 & 14 \end{bmatrix}$$

   Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution $\mathbf{x} \in \mathbb{R}^3$ for every $\mathbf{b} \in \mathbb{R}^3$?

4. Write the augmented matrix for the linear system that corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$, Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -5 & -1 \\ 0 & 2 & 3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

5. Let

$$A = \begin{bmatrix} 1 & 3 & 0 & 4 & 5 \\ 2 & 6 & 1 & 7 & 8 \\ 3 & 9 & 0 & 10 & 11 \end{bmatrix},$$

   Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form

6. Suppose A is a 3×6 matrix with 3 pivot positions.
   (a) Does the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?
   (b) Does the equation $A\mathbf{x} = \mathbf{b}$ have at least one solution for every $\mathbf{b} \in \mathbb{R}^3$?

7. Suppose A is a 6×3 matrix with 3 pivot positions.
   (a) Does the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?
   (b) Does the equation $A\mathbf{x} = \mathbf{b}$ have at least one solution for every $\mathbf{b} \in \mathbb{R}^6$?