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Math 2331 Homework Solutions

Hw #2

1. $Ax = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 7 & -8 & 9 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1-4+9 \\ -4+10-18 \\ 7-16+27 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \\ 18 \end{bmatrix}$

④ or $1\begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 5 \\ -8 \end{bmatrix} + 3\begin{bmatrix} 3 \\ -6 \\ 9 \end{bmatrix} = \begin{bmatrix} 1-4+9 \\ -4+10-18 \\ 7-16+27 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \\ 18 \end{bmatrix}$

2. a. Is $b_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ in Span$\{v_1, v_2, v_3\}$? $v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ $v_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ $v_3 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$

Is there a solution $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ to $b_1 = x_1 v_1 + x_2 v_2 + x_3 v_3$

⑧ $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & -3 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \rightarrow$ Augmented matrix $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 3 & 0 & -3 & -1 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -6 & -12 & -4 \\ 0 & -3 & -6 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 2/3 \\ 0 & -3 & -6 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 2/3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

System is inconsistent, b_1 is not in Span$\{v_1, v_2, v_3\}$

b. Is b_2 in Span$\{v_1, v_2, v_3\}$? $x_1 v_1 + x_2 v_2 + x_3 v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$?

Augmented matrix

⑧ $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 3 & 0 & -3 & 1 \\ 2 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -6 & -12 & -2 \\ 0 & -3 & -6 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 1/3 \\ 0 & 3 & 6 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & -1 & 1/3 \\ 0 & 1 & 2 & 1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $x_1 = 1/3 + x_3$
$x_2 = 1/3 - 2x_3$

System is consistent; $\begin{bmatrix} x_1 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \overset{?}{=} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ✓

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in$ Span$\{v_1, v_2, v_3\}$, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1/3\, v_1 + 1/3\, v_2$

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3. Does $\begin{bmatrix} 1 & -2 & 4 \\ 1 & -1 & 3 \\ 1 & 3 & 14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = b$ have a solution for every $b \in \mathbb{R}^3$?
$\quad A \qquad\qquad x$

Solution Find an echelon form for A.

⑥

$$A \sim \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -1 \\ 0 & 5 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 15 \end{bmatrix}$$

A has a pivot position in every row, so $Ax=b$ has a solution for every $b \in \mathbb{R}^3$.

4. Write the augmented matrix for $\begin{bmatrix} 1 & 2 & 1 \\ -3 & -5 & -1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Solve, write solution as vector.

Solution Augmented matrix: $[A \ b] = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -5 & -1 & 1 \\ 0 & 2 & 3 & 1 \end{bmatrix}$

⑧

$$\to \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 3 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 1 \quad x_2 = -1, \ x_3 = 1$

Check $1 \cdot \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} - 1 \cdot \begin{bmatrix} 2 \\ -5 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}}$$

5. $Ax = 0$ with $A = \begin{bmatrix} 1 & 3 & 0 & 4 & 5 \\ 2 & 6 & 1 & 7 & 8 \\ 3 & 9 & 0 & 6 & 11 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 0 & 4 & 5 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & -2 & -4 \end{bmatrix}$

⑧

$$\to \begin{array}{c} \begin{bmatrix} 1 & 3 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \\ \begin{matrix} p & F & p & p & F \end{matrix} \end{array}$$

$x_1 = 3x_5 - 3x_2$

$x_3 = 0$

$x_4 = -2x_5$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3x_2 + 3x_5 \\ x_2 \\ 0 \\ -2x_5 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 3 \\ 0 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

x_2, x_5 free variables

x_1, x_3, x_4 basic variables

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<u>6.</u> A 3×6 with 3 pivot positions

<u>a.</u> Does $Ax=0$ have a nontrivial solution?

④ <u>Yes</u> $Ax=0$ has 3 free variables.

④ b. Does $Ax=b$ have a solution for every $b \in \mathbb{R}^3$?

Yes. A has a pivot position in every row.

<u>7.</u> A is 6×3 with 3 pivot positions

④ <u>a.</u> Does $Ax=0$ have a nontrivial solution?

No. There are no free variables

<u>b.</u> Does $Ax=b$ have a solution for every $b \in \mathbb{R}^6$?

④ No Some rows have no pivot.