Math 2331
January 30, 2020 Homework 3 Name ________________

1. Determine if these vectors are linearly independent. Justify your answer.

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, \; \mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \; \mathbf{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

2. Determine if these vectors are linearly independent. Justify your answer.

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, \; \mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}.$$

3. Determine if these vectors are linearly independent. Justify your answer.

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \; \mathbf{v}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \; \mathbf{v}_3 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

4. Determine if these vectors are linearly independent. Justify your answer.

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, \; \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \; \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and maps $\mathbf{v} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$ into $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$.
   Use the fact that T is linear to find:
   (a) $T(3\mathbf{u})$
   (b) $T(\mathbf{u} - 2\mathbf{v})$
   (c) $T(2\mathbf{u} + \mathbf{v})$.

6. A homogeneous linear system of equations has a coefficient matrix A which is row equivalent to the following matrix R in reduced echelon form:

$$R = \begin{bmatrix} 1 & 5 & 0 & 2 & 0 & 5 \\ 0 & 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -3 \end{bmatrix}$$

   (a) Describe the solution set in parametric vector form (See p. 44-45).
   (b) Find the solution with $x_2 = 1,\ x_4 = 0,\ x_6 = 0$.
   (c) Suppose the first column of A is:

$$\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

   What is the second column of A? *Hint:* Use the answer to part b.
   (d) Find the solution with $x_2 = 0,\ x_4 = 1,\ x_6 = 0$.

(e) If, in addition to the above, the third column of A is

$$\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix},$$

what is the fourth column of A? *Hint:* Use the answer to part d.

(f) Are the pivot columns of R linearly independent, or linearly dependent? What (if anything) does this mean for the pivot columns of A?

7. Let $T : \mathbb{R}^n \to \mathbb{R}^m$, and suppose $\{\mathbf{v}_1, \cdots, \mathbf{v}_p\}$ is a set of vectors in $\mathbb{R}^n$ such that $\{T(\mathbf{v}_1), \cdots, T(\mathbf{v}_p)\}$ is linearly independent. Show that $\{\mathbf{v}_1, \cdots, \mathbf{v}_p\}$ is linearly independent.

8. Find the standard matrix of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first rotates points through $\pi/3$ radians (counterclockwise) and then performs a horizontal shear that transforms $\mathbf{e}_2$ into $\mathbf{e}_2 + 3\mathbf{e}_1$ and which leaves $\mathbf{e}_1$ unchanged.

9. Show that if the columns of AB are linearly independent, then the columns of B are linearly independent.