EXERCISES FOR MATH 2331 DUE FEBRUARY 11

(1) Let A be an $n \times n$ matrix. Show that if A is not invertible, then the columns of A are linearly dependent.

(2) Use the algorithm on page 110 to find the inverse of this matrix, if it exists:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

(3) Determine which of these matrices is invertible. For any that are invertible, find the inverse.

(a) $A = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 7 & 15 \\ 3 & 10 & 23 \end{bmatrix}$.

(b) $B = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 7 & 15 \\ 3 & 10 & 22 \end{bmatrix}$.

(4) Find a basis for Col(A), and a basis for Nul(A), if $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -6 & 4 \\ 4 & 1 & 5 & 14 \end{bmatrix}$.

(5) Let $\beta = \{(1,3)\,,\ (3,2)\}$. Please accept β as a basis for $\mathbb{R}^2$ and find the coordinate vector $[\mathbf{x}]_\beta$, if $\mathbf{x} = (-2,\ 5)$.

(6) For a certain 2×5 matrix A, the system $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b} \in \mathbb{R}^2$. What is the dimension of Nul(A)?

(7) If A is a 12×8 matrix, how large could $\dim(Nul(A))$ be?

(8) Use the algorithm on page 110 to find the inverse of this matrix, if it exists:

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$