EXERCISES FOR MATH 2331 DUE FEBRUARY 19

(1) Let $D_1 = -2$, $D_2 = \det \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$, $D_3 = \det \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$, $D_4 = \det \begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$, etc. Use a cofactor expansion to find a formula for D_{n+1} in terms of D_n and D_{n-1}. Calculate D_4, D_5 and D_{10} . Can you prove a formula for D_n which does not use other $D's$?

(2) Let $\gamma = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ and let β be the standard basis of $\mathbb{R}^2$.

(a) Suppose $[\mathbf{x}]_\gamma = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$. Find $[\mathbf{x}]_\beta$.

(b) Suppose $[\mathbf{y}]_\beta = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$. Find $[\mathbf{y}]_\gamma$.

(3) Find the determinant of $A = \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{bmatrix}$.