Math 2331 Homework Solutions

HW #5

1) $D_1 = -2$. $D_2 = \det\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix}$, $D_3 = \det\begin{bmatrix}-2 & 1 & 0\\ 1 & -2 & 1\\ 0 & 1 & -2\end{bmatrix}$, $D_4 = \begin{vmatrix}-2 & 1 & 0 & 0\\ 1 & -2 & 1 & 0\\ 0 & 1 & -2 & 1\\ 0 & 0 & 1 & -2\end{vmatrix}$, etc

By cofactor expansion: $D_{n+1} = -2 D_n - 1\cdot\det\begin{bmatrix}1 & 1 & 0 & \cdots\\ 0 & -2 & 1 & 0 & \cdots\\ 0 & 1 & -2 & 1 & \cdots\end{bmatrix} =$

$-2D_n - D_{n-1}$.

(10)

$D_2 = 4-1 = 3$, $D_3 = -2D_2 - D_1 = -6+2 = -4$

$D_4 = -2D_3 - D_2 = 8-3 = 5$

$D_5 = -2D_4 - D_3 = -10-(-4) = -6$

$D_6 = -2D_5 - D_4 = 12-5 = 7$

$\cdots$ $D_n = (-1)^n(n+1)$, $D_{10} = 11$.

Prove $D_n = (-1)^n(n+1)$ without other D's?

Induction. True for $D_3 = -4$, $D_2 = 3$

Suppose true for D_n, D_{n-1}

Then $D_{n+1} = -2D_n - D_{n-1}$

$= (-2)(-1)^n(n+1)$

$- (-1)^{n-1}n$

$= (-1)^n(-2(n+1)+n)$

$= (-1)^n(-n-2)$ ✓

$= (-1)^{n+1}(n+2)$

2. $\gamma = \left\{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}\right\}$ β = std basis $\mathbb{R}^2$

a. If $[x]_\gamma = \begin{bmatrix}5\\7\end{bmatrix}$ find $[x]_\beta$.

(5) **Solution** $[x]_\beta = 5\begin{bmatrix}2\\1\end{bmatrix} + 7\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}10+7\\5+21\end{bmatrix} = \begin{bmatrix}17\\26\end{bmatrix}$

b. If $[y]_\beta = \begin{bmatrix}4\\-5\end{bmatrix}$ find $[y]_\gamma$

(8) **Solution** Solve $\begin{bmatrix}4\\-5\end{bmatrix} = y_1\begin{bmatrix}2\\1\end{bmatrix} + y_2\begin{bmatrix}1\\3\end{bmatrix}$. Then $[y]_\gamma = \begin{bmatrix}y_1\\y_2\end{bmatrix}$

Augmented Matrix $\begin{bmatrix}2 & 1 & 4\\ 1 & 3 & -5\end{bmatrix} \to \begin{bmatrix}1 & 3 & -5\\ 2 & 1 & 4\end{bmatrix} \to \begin{bmatrix}1 & 3 & -5\\ 0 & -5 & 14\end{bmatrix}$

$\to \begin{bmatrix}1 & 3 & -5\\ 0 & 1 & -\frac{14}{5}\end{bmatrix} \to \begin{bmatrix}1 & 0 & \frac{17}{5}\\ 0 & 1 & -\frac{14}{5}\end{bmatrix}$ $y_1 = \frac{17}{5}$, $y_2 = -\frac{14}{5}$

$[y]_\gamma = \begin{bmatrix}-3/5\\4/5\end{bmatrix}$ Check $\frac{17}{5}\begin{bmatrix}2\\1\end{bmatrix} - \frac{14}{5}\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}\frac{34-14}{5}\\ \frac{17-42}{5}\end{bmatrix} = \begin{bmatrix}4\\-5\end{bmatrix}$ ✓

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3. $\det\begin{bmatrix}1 & 3 & 2 & -4\\ 0 & 1 & 2 & -5\\ 2 & 7 & 6 & -3\\ -3 & -10 & -7 & 2\end{bmatrix} \overset{R_3-2R_1\to R_3,\ R_4+3R_1\to R_4}{=} \det\begin{bmatrix}1 & 3 & 2 & -4\\ 0 & 1 & 2 & -5\\ 0 & 1 & 2 & 5\\ 0 & -1 & -1 & -10\end{bmatrix} \overset{R_3-R_2\to R_3}{=} \det\begin{bmatrix}1 & 3 & 2 & -4\\ 0 & 1 & 2 & -5\\ 0 & 0 & 0 & 10\\ 0 & 0 & 1 & -10\end{bmatrix}(-1)$

(10)

$= -\det\begin{bmatrix}1 & 3 & 2 & -4\\ 0 & 1 & 2 & -5\\ 0 & 0 & 0 & 10\\ 0 & 0 & -1 & 15\end{bmatrix} \overset{-\det}{=} \begin{bmatrix}1 & 3 & 2 & -4\\ 0 & 1 & 2 & -5\\ 0 & 0 & -1 & 15\\ 0 & 0 & 0 & 10\end{bmatrix} \times(-1) = -10$

$R_4 - R_2 \to R_4$ exch R_3, R_4