

Show all work!

In questions 1-4, determine if the given set is a subspace of \mathbb{P}_2 . Justify your answers.

1. All polynomials p in \mathbb{P}_2 such that $p(1) = 0$.
2. All polynomials p in \mathbb{P}_2 such that $p(0) = 1$.
3. All polynomials p in \mathbb{P}_2 such that $p'(1) = 0$.
4. All polynomials p in \mathbb{P}_2 such that $p'(0) = 1$.
5. Show that the set of 2×2 matrices A such that $A \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is a vector subspace of $M_{2 \times 2}$, the vector space of 2×2 matrices.
6. Let H be the kernel of the linear transformation $T : \mathbb{P}_2 \rightarrow \mathbb{R}^2$, $T(p) = \begin{bmatrix} p(1) \\ p(2) \end{bmatrix}$. Find a basis for H . Then find the dimension of the range of T .