Show all work!

1. Find a basis and the dimension for this subspace of $\mathbb{R}^4$: $\left\{ \begin{bmatrix} 2a+6b-c \\ 4a-3b-2c \\ -2a-6b+c \end{bmatrix} : a,\ b,\ c \in \mathbb{R} \right\}$.

   Is this set more naturally the column space of a matrix, the row space of a matrix, or the null space of a matrix?

2. Find a basis and the dimension for this subspace of $\mathbb{R}^3$:

$$\{(a,\ b,\ c) : a - 5b + 2c = 0, b - 3c = 0, a - 4b - c = 0\}. \tag{1}$$

   Is this set more naturally the column space of a matrix, the row space of a matrix, or the null space of a matrix?

3. Assume that the matrix A is row equivalent to B. Without calculation, list rank(A) and $\dim(NulA)$. Then find bases for $ColA$, $RowA$, and $NulA$.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 5 & 7 & 9 & 11 \\ 1 & 1 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 5 & 7 & 9 \\ 2 & 3 & 4 & 6 & 8 & 10 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

4. Let $H = \{p(t) \in \mathbb{P}_4 : p'(0) = 0,\ p(0) = 0\}$.

   (a) Show that H is a vector subspace of $\mathbb{P}_4$.

   (b) Find a basis and the dimension of H.

   (c) Let $T : H \to \mathbb{P}_2$, $T(p(t)) = p''(t)$. Please accept that T is a linear transformation. What is its rank?

5. Let $M = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$. Find a steady state vector for M.