EXERCISES FOR MATH 2331 DUE APRIL 8

(1) Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ \frac{1}{2} & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -4 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & 2 \\ 0 & 0 & 1 \\ -\frac{1}{2} & 1 & -2 \end{bmatrix} = PDP^{-1}.$$

Use the Diagonalization Theorem to find the eigenvalues of A and a basis for each eigenspace.

(2) Diagonalize $A = \begin{bmatrix} -4 & -1 \\ 1 & -2 \end{bmatrix}$ or explain why it is not diagonalizable.

(3) Diagonalize $A = \begin{bmatrix} -4 & 2 \\ 1 & -2 \end{bmatrix}$ or explain why it is not diagonalizable.

(4) Suppose $\mathbf{v},\ \mathbf{u},\ \mathbf{w}$ are vectors in $\mathbb{R}^n$ such that $\|\mathbf{v}\| = 4$, $\|\mathbf{u}\| = 3$, $\|\mathbf{w}\| = 6$, $\mathbf{v} \cdot \mathbf{u} = 10$, $\mathbf{v} \cdot \mathbf{w} = 7$, $\mathbf{u} \cdot \mathbf{w} = -2$.

(a) Find $\|\mathbf{v} + \mathbf{u}\|$.

(b) Find the projection of $\mathbf{w}$ onto the span of $\mathbf{u}$.

(5) Let $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$. Verify that $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ is an orthogonal basis for $\mathbb{R}^2$ and express $\mathbf{u}$ as a linear combination of $\{\mathbf{v}_1,\ \mathbf{v}_2\}$.