Find the Jacobian of the transformation:

1. $x = aAu + Bv, \quad y = Cu + Dv$: $dx\ dy = J\ du\ dv$. $J = ?$.
2. θ fixed, $x = u\cos(\theta) - v\sin(\theta), \quad y = u\sin(\theta) + v\cos(\theta)$. (rotation by θ) $dx\ dy = J\ du\ dv$. $J = ?$
3. $x = uv, \quad y = u^2 + v^2$. $dx\ dy = J\ du\ dv$. $J = ?$
4. Let $\Omega = \{(x, y)\ 0 \leq x - 3y \leq 2, \quad 0 \leq x + 3y \leq 2\}$. Evaluate $\int\int_\Omega x^2 - 9y^2\ dx\ dy$.
5. Integrate $\mathbf{h}(x, y) = xy\mathbf{i} + x\mathbf{j}$ over the path $\mathbf{r}(t) = t\mathbf{i} + (2 - 2t)\mathbf{j}$, $0 \leq t \leq 1$.
6. Integrate $\mathbf{h}(x, y) = xy\mathbf{i} + x\mathbf{j}$ over the path $\mathbf{r}(t) = t\mathbf{i} + (2 - 2t^2)\mathbf{j}$, $0 \leq t \leq 1$.
7. Integrate $\mathbf{h}(x, y) = (2xy^2 - x)\,\mathbf{i} + (2x^2y + 3x)$ counterclockwise around the boundary of the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 3$.
8. An object moves along the path $\mathbf{r}(t) = 2t\mathbf{i} + (1 - t)^2\mathbf{j}$, $0 \leq t \leq 1$. One of the forces acting on the object is $\mathbf{F} = 3x^2y\mathbf{i} + x^3\mathbf{j}$. Find the work done by $\mathbf{F}$.

Determine whether $\mathbf{h}$ is a gradient. If $\mathbf{h}$ is a gradient, calculate the indicated line integral.

9. $\mathbf{h}(x, y) = (\sin(x) + 2y)\,\mathbf{i} + (\cos(y) + 2x)\,\mathbf{j}$. $\mathbf{r}(t) = 3\cos(t)\mathbf{i} + 4\sin(t)\mathbf{j}$, $0 \leq t \leq 2\pi$.
10. $\mathbf{h}(x, y) = (\sin(x) + 2y)\,\mathbf{i} + (\cos(y) - 3x)\,\mathbf{j}$. $\mathbf{r}(t) = 3\cos(t)\mathbf{i} + 4\sin(t)\mathbf{j}$, $0 \leq t \leq 2\pi$.
11. Evaluate $\oint_C y^2 dx + xy\ dy$ counter-clockwise around the boundary of the triangle $0 \leq x \leq 2$, $0 \leq y \leq \frac{3x}{2}$. *Hint:* Use Green's Theorem.
12. Let C be a piecewise-smooth Jordan curve that does not pass through the origin. Evaluate

$$\oint_C \frac{x\ dy}{x^2 + y^2} - \frac{y\ dx}{x^2 + y^2} \text{ counter-clockwise :} \tag{1}$$

   (a) If C does not enclose the origin.
   (b) If C encloses the origin, compare this integral to the same integral over a circle of small radius.