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1. Let $f(x, y) = 7x^2y - 5y^2 - 3x$. Find the directional derivative of f at $(1, 2)$ in the direction of $3\mathbf{i} + 4\mathbf{j}$.
2. Let $f(x, y) = 3x^2 - 2xy + y^2$.
 - (a) Find a unit vector \mathbf{u} that maximizes the directional derivative of f at $(2, 3)$ in the direction \mathbf{u} .
 - (b) Find the maximum directional derivative of f at $(2, 3)$.
3. Let $f(t) = \int_0^{t^2} e^{x^2} dx$. Find $f'(2)$. *Hint: Use the Fundamental Theorem of Calculus*
4. Let $f(x, y) = \int_y^x e^{t^2} dt$. Find $\nabla f(x, y)$.
5. Let g be continuous on \mathbb{R} , let c be a positive constant and let $u(x, t) = \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) ds$.
 - (a) Show that for $t > 0$, $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$.
 - (b) Show that $\lim_{t \rightarrow 0^+} \frac{\partial u}{\partial t}(x, t) = g(x)$.
6. Find $\frac{du}{dt}$ if $u = e^x \cos y$ and $x = t^2$, $y = \pi t$.