Show all work!

1. Suppose $z = f(x, y)$ is continuously differentiable, $f(1, 4) = 3$ and $\frac{\partial f}{\partial x}(1, 4) = 5$, $\frac{\partial f}{\partial y}(1, 4) = 2$. Then there is a function $x = g(y, z)$ defined in a neighborhood of $(y, z) = (4, 3)$ with $g(y, z)$ continuously differentiable. Fiind $\frac{\partial g}{\partial y}(4, 3)$
Hint: Start with the plane tangernt to the graph of f at $(x, y, z) = (1, 4, 3)$ and solve for $x - 1$ (or dx).

2. Find the minimum and maximum values of xy on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Also, identify the points on the ellipse at which the minimum and maximum values occur.

3. Determine which of these two vector fields is the gradient of a function $f(x, y)$. For the vector field that is a gradient, find all functions $f(x, y)$ such that $\nabla f(x, y)$ is the given vector field.

   (a) $\mathbf{v}(x, y) = 2xe^{3y}\mathbf{i} + (3x^2e^{3y} + 5)\,\mathbf{j}$.

   (b) $\mathbf{u}(x, y) = 2ye^{3x}\mathbf{i} + (3y^2e^{3x} + 5)\,\mathbf{j}$.