

Find the Jacobian of the transformation:

1. $x = aAu + Bv, \quad y = Cu + Dv: dx dy = J du dv. \quad J = ?.$

2. θ fixed, $x = u \cos(\theta) - v \sin(\theta), \quad y = u \sin(\theta) + v \cos(\theta).$ (rotation by θ) $dx dy = J du dv. \quad J = ?$

3. $x = uv, \quad y = u^2 + v^2. dx dy = J du dv. \quad J = ?$

4. $x = uv^2, \quad y = u^2v, dx dy = J du dv. \quad J = ?$

5. $x = u - \ln v, \quad y = \ln u + v. dx dy = J du dv. \quad J = ?$

6. Cylindrical coordinates:

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z. \quad (1)$$

$$dx dy dz = J dr d\theta dz. \quad J = ?$$

7. Spherical coordinates:

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi. \quad (2)$$

$$dx dy dz = J d\rho d\phi d\theta. \quad J = ?$$

In the next exercises, let $\Omega = \{(x, y) \mid 0 \leq x - 3y \leq 2, \quad 0 \leq x + 3y \leq 2\}.$

9. Evaluate $\int \int_{\Omega} x^2 - 9y^2 dx dy.$