

1. Assume the following are defined:
- Neighborhood
 - Interior point
 - Boundary point

Define the following concepts. Use complete sentences and be sure to identify the object being defined.

- Open (5 pts each)
- Closed
- Limit point
- Compact
- Continuous at a point x_0
- Continuous on a set D

2. Prove that a union of an infinite collection of open sets is open. 10 pts

3. Prove that if K is compact and $f:K \rightarrow \mathbf{R}^n$ is continuous, then $f(K)$ is compact. 15 pts

3. Prove that there is a solution to $e^x = \frac{3}{x^2}$ between $x = 1$ and $x = 2$. You may assume that the exponential function is continuous and that e is approximately 2.72. 15 pts

4. Use the definition of limit to show that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2 - 3y^2}{\sqrt{x^2 + y^2}} = 0. \quad 15 \text{ pts}$$

5. Let Q_0 be the set of rational numbers in $[0,1]$. Prove that the closure of Q_0 is $[0,1]$. 15 pts