Math 3334
October 28, 2019 Exam 2 Review

1. Suppose that f is a function on $\mathbb{R}$ such that $|f(b) - f(a)| \leq M\,|b-a|^2$ for all $a,\ b \in \mathbb{R}$. Prove that f is a constant function. (20)

2. Define *Lipschitz continous.* (8)

3. Let U be a convex open set in $\mathbb{R}^2$ and let $f : U \to \mathbb{R}$ be differentiable on U with $\|Df(x,y)\| \leq 5$ for all $(x,y) \in U$. Prove that f is Lipschitz continuous on U. (17)

4. Let $f(x,y) = \int_0^{x+2y} e^{-t^2}\ dt$.
   (a) Find $Df(x,y)$ (10)

   (b) Find the directional derivative of f at $(3,1)$, $D_\beta f(3,1)$, where $\beta = (3,4)/5$. (10)

   (c) Find a unit vector β which maximizes $D_\beta f(3,1)$, and find the maximum value. (10)

5. Suppose f is a C^1 function on an open set U in $\mathbb{R}^2$, and $z = f(x,y)$, $Df(x,y) = (p(x,y),\ T(x,y))$. If $x = g(y,z)$, find $Dg(y,z)$. What simple condition on p and/or T is necessary for this computation? (25)