Math 3334 **FINAL EXAM** **NAME________________**
May 11, 2005 **ID #________________**

1. a. Let f be differentiable on R^2, and let $g(r,\theta) = f(r\cos(\theta), r\sin(\theta))$.

Find a vector equation of the form $\nabla g = \nabla f \cdot A(r,\theta)$, where $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$, $\nabla g = \left(\frac{\partial g}{\partial r}, \frac{\partial g}{\partial \theta}\right)$, and $A(r,\theta)$ is a 2x2 matrix.

14 pts

b. Use the vector equation found in part a. to find $B(r,\theta)$ such that $\nabla f = \nabla g \cdot B(r,\theta)$. Interpret in terms of the unit vectors $\mathbf{e}_r, \mathbf{e}_\theta$ in the r and θ directions. 12 pts

2. Use the definition of limit to show that
$\lim_{(x,y)\to(1,2)} 2y^2 - 3x^2 = 5$. 16 pts

3. Determine whether the solution set of the equation
$xy + \cos(xyz) + z^2 = 3$
has the form:
   a. z = f(x,y) near (1,2,0). If so, compute $\nabla f(1,2)$ 10 pts

   b. x = g(y,z) near (1,2,0). If so, compute $\nabla g(2,0)$ 10 pts

   c. y = h(x,z) near (1,2,0). If so, compute $\nabla h(1,0)$ 10 pts

4. Suppose $\mathbf{a}$ and $\mathbf{b}$ are vectors in R^n such that:
$\|\mathbf{a}\| = 5$, $\|\mathbf{b}\| = 6$
   i. Find the range of possible values of $\mathbf{a} \cdot \mathbf{b}$ 10 pts

   ii. Suppose $\mathbf{a} \cdot \mathbf{b} = 3$. Find $\|\mathbf{a} - \mathbf{b}\|$, and $\cos(\theta)$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. 16 pts

5. Let $f : A \to R$ be continuous on a bounded open region $A \subset R^n$. Prove that if $\int_A |f|\, dV = 0$, then $f(x) = 0$ for all $x \in A$. 16 pts

6. Change the order of integration, and evaluate: 14 pts
   a. $\int_0^2 \int_{y/2}^1 (x+y)^2 \, dx\, dy$

   b. $\int_0^{\pi/2} \int_0^{\sin(x)} \cos(x)\, dy\, dx$ 14 pts

7. Let x(t) denote the position of an object in $\Re^3$ at time t. Suppose that the speed $\|x'(t)\|$ is constant and non-zero. Prove that the acceleration $x''(t)$ is always orthogonal to the velocity.

14 pts

8. Determine which of the following vector fields is the gradient of a function $f(x,y)$. If it is, find all such functions $f(x,y)$.

24 pts

a. $\mathbf{V}(x,y) = \left(3xy^2 + 3y\right)\mathbf{i} + \left(x^3 + xy^2 - 7y\right)\mathbf{j}$

b. $\mathbf{W}(x,y) = \left(3x^2y + y^3\right)\mathbf{i} + \left(x^3 + 3xy^2 + 6y\right)\mathbf{j}$