A Robust Full Waveform Inversion Scheme for Vector-Acoustic Seismic Data using Theory of Dirichlet-Neumann Type Elliptic Inverse Problems

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ABSTRACT

Full waveform inversion is widely regarded as the most accurate method of obtaining acoustic wave velocity model in seismic exploration. Recent advances in seismic acquisition technology allow for the recording of vector-acoustic data i.e. pressure field and its multi component gradient.

In order to obtain an improved description of wave propagation, we extend the acoustic equation to include the sub-surface density and attenuation terms in addition to velocity. By incorporating measurements of the normal derivative of pressure at the surface, and through a finite Laplace transform, we reformulate the inverse problem of recovering parameters ρ, κ and γ from the hyperbolic acoustic equation

$$\begin{aligned} \frac{1}{\rho(x)\kappa^2(x)}(p_{tt} + \gamma(x)p_t) - \nabla \cdot \frac{1}{\rho(x)}\nabla p &= F(x,t) \qquad \text{on } \Omega \\ p(x,t) = \zeta(x,t) &\qquad \text{on } \Gamma \\ \frac{\partial p}{\partial n} = \chi(x,t) &\qquad \text{on } \Gamma \end{aligned}$$

to a problem of recovering parameters A, B, C, R and S from the elliptic equation

$$-\nabla \cdot (A(x)\nabla u) + (\lambda^2(x)B(x) + \lambda C(x))u = -e^{-\lambda}(\lambda R(x) + S(x)) + \hat{F}(x,\lambda)$$

with a known Dirichlet and partially known Neumann data. The coefficients A, B, C, R & S of the transformed elliptic equation are then recovered as the (conjectured) unique stationary points of an appropriate functional by using Neuberger gradient descent.

As a special case, we apply our algorithm to the electrical impedance tomography problem of recovering (discontinuous) electrical conductivity. The use of Neumann Neuberger gradient allow us to weaken the conductivity assumption on the boundary.

For the full waveform inversion problem our approach yields a fully non linear technique capable of stably recovering multiple coefficients from boundary measurements.

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