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Orthogonal Bases and Reproducing Kernels for Hilbert Spaces of Harmonic Functions

The space of finite energy H^1-harmonic functions on a region Ω has a basis of Steklov eigenfunctions that are L^2-orthogonal on the boundary and have orthogonal gradients on the region. This leads to a constructive characterization of the boundary trace spaces $H^{1/2}(\partial\Omega)$ for the region with explicit formulae for inner products and some standard operators. In particular there are general formulae for the Poisson kernel and for the solution operators for Robin and Neumann boundary value problems on the region Ω.

This analysis is extended to a 1-parameter family of trace spaces $H^s(\partial\Omega)$ and their harmonic extensions $\mathcal{H}^{s+1/2}(\Omega)$. We shall show that, provided the boundary data is L^2, the corresponding spaces of harmonic functions on Ω are reproducing kernel Hilbert spaces with respect to a natural inner product. The reproducing kernels are found explicitly via the Steklov eigenfunctions of the domain. This extends earlier results of Peetre et al and J.L. Lions.