**Abstract.** The main goal of this lecture is to discuss the numerical solution of the following *inverse problem* (from Geophysics) for the *Eikonal equation*:

\[
\begin{aligned}
\text{Find } & c_{\text{opt}} \in \mathcal{C}, \text{ such that } \\
J(c_{\text{opt}}) & \leq J(c), \forall c \in \mathcal{C},
\end{aligned}
\]

where

\[
\mathcal{C} = \{ c \mid c \in L^\infty(\Omega), 0 < c_{\text{min}} \leq c \leq c_{\text{max}} \},
\]

\[
J(c) = \frac{1}{2} \int_{\Gamma} | z - y_{\text{meas}} |^2 d\Gamma,
\]

\(z\) (the time of first arrival) being the solution of the following Eikonal problem

\[
\begin{aligned}
|\nabla z| &= \frac{1}{c} \text{ in } \Omega, \\
z &\geq 0, \\
z(x_0) &= 0.
\end{aligned}
\]

Above, \(\Omega \subset \mathbb{R}^d (d \geq 2), \Gamma \subset \overline{\Omega}, x_0 \in \overline{\Omega}\) being the point source of the wave. In practice, several sources are used to reconstruct \(c_{\text{opt}}\).

In order to solve the above inverse problem, we advocate a methodology combining external penalty, harmonic and bi-harmonic regularizations, operator-splitting to decouple nonlinearities and differential operators, and finite element methods. The results of some numerical experiments will be presented.