Outline

1. Probability
2. Counting Techniques
3. Probability
An **probability measure** is a function which assign numbers between 0 and 1 to any event in the sample space $\Omega$.

If the sample space $\Omega$, the collection of events, and the probability measure are all specified, they constitute a **probability model** of the random experiment.
Assigning probabilities

- **Classical method** is used when all the experimental outcomes are equally likely. If \( n \) experimental outcomes are possible, a probability of \( 1/n \) is assigned to each experimental outcome. Example: Drawing a card from a standard deck of 52 cards. Each card has a 1/52 probability of being selected.

- **Relative frequency method** is used when assigning probabilities is appropriate when data are available to estimate the proportion of the time the experimental outcome will occur if the experiment is repeated a large number of times. That is for any outcome, \( A \), probability of \( A \) is

\[
P(E) = \frac{\text{number of times } E \text{ occurs}}{\text{total number of observations}} = \frac{\#(E)}{N}
\]

- \( P(E) \) is a probability model for any event \( E \) that is a subset of \( \Omega \).
Example of Probabilities

Relative frequency method: An insurance company determined the number of accidents in a year. A sample of 100 people were surveyed to determine the number of accidents they were in a year: 0 accidents 25 people, 1 accident 45 people, 2 accidents 20 people, 3 or more accidents 10 people. The following table shows the relative frequency for the outcomes.

<table>
<thead>
<tr>
<th>Number of accidents</th>
<th>Frequency (count)</th>
<th>Relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25</td>
<td>(\frac{25}{100} = 0.25)</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>(\frac{45}{100} = 0.45)</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>(\frac{20}{100} = 0.20)</td>
</tr>
<tr>
<td>3 or more</td>
<td>10</td>
<td>(\frac{10}{100} = 0.10)</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>1</td>
</tr>
</tbody>
</table>
What is the probability of getting a sum of 5?
How to get $\#(E)$

- $\#(E)$ denotes the number of elements in the subset $E$.
- Sometimes it is not obvious as the previous example. Thus we need to use some counting techniques to determine $\#(E)$. 
In the city of Milford, applications for zoning changes go through a two-step process:

1. A review by the planning commission.
2. A final decision by the city council.

- At step 1 the planning commission reviews the zoning change request and makes a positive or negative recommendation concerning the change.
- At step 2 the city council reviews the planning commission’s recommendation and then votes to approve or to disapprove the zoning change.

How many possible decisions can be made for a zoning change in Milford?
Counting Rules

- If an experiment can be described as a sequence of $k$ steps with $n_1$ possible outcomes on the first step, $n_2$ possible outcomes on the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2)\ldots(n_k)$.

- A tree diagram can be used as a graphical representation in visualizing a multiple-step experiment.
Tree diagram

Step 1
Planning Commission

Step 2
City Council

Sample Points

(positive, approve)

(positive, disapprove)

(negative, approve)

(negative, disapprove)

positive

disapprove

approve

da

negative
Examples

1. How many ways can we select 4 digits and 3 letters?
   - If digits and letters are allowed to repeat?
   - If digits are letters are not allowed to repeat?

2. In how many ways can 4 people be seated in 6 seats?
Allowing Repeated Values

When we allow repeated values, the number of orderings of $n$ objects taken $r$ at a time, with repetition is $n^r$.

Example 3: In how many ways can you write 4 letters on a tag using each of the letters C O U G A R with repetition?
Permutations

It allows one to compute the number of outcomes when \( r \) objects are to be selected from a set of \( n \) objects where the order of selection is important. The number of permutations is given by

\[
P_r^n = \frac{n!}{(n - r)!}
\]

- Where \( n! = n(n - 1)(n - 2) \cdots (2)(1) \)
- Rocode for \( n! \): factorial(n)
Counts the number of experimental outcomes when the experiment involves selecting $r$ objects from a (usually larger) set of $n$ objects. The number of combinations of $n$ objects taken $r$ unordered at a time is

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Rcode: `choose(n,r)`
Examples

4. In how many ways can a committee of 5 be chosen from a group of 12 people?

5. In a manufacturing company they have to choose 5 out of 50 boxes to be sent to a store. How many ways can they choose the 5 boxes?
Example 6

From a committee of 10 people.

a) In how many ways can we choose a chair person, a vice-chair person, and a secretary, assuming that one person cannot hold more than one position?

b) In how many ways can we select a subcommittee of 3 people?
Example

7. A researcher randomly selects 3 fish from a tank of 12 and puts each of the 3 fish into different containers. How many ways can this be done?

8. Among 10 electrical components 2 are known not to function. If 5 components are randomly selected, how many ways can we have only one of components not functioning?
Assigning probabilities

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- **Relative frequency method** is used when assigning probabilities is appropriate when data are available to estimate the proportion of the time the experimental outcome will occur if the experiment is repeated a large number of times. That is for any event $E$, probability of $E$ is

\[
P(E) = \frac{\text{number of times } E \text{ occurs}}{\text{total number of observations}} = \frac{n(E)}{n(S)}\]
Example 1

If 5 marbles are drawn at random all at once from a bag containing 8 white and 6 black marbles, what is the probability the 2 will be white and 3 will be black?
Example 2

The qualified applicant pool for six management trainee positions consists of seven women and five men.

1. What is the probability that a randomly selected trainee class will consist entirely of women?

2. What is the probability that a randomly selected trainee class will consist of an equal number of men and women?
Example 3

Suppose a box contains 3 defective light bulbs and 12 good bulbs. Suppose we draw a simple random sample of 4 light bulbs, find the probability that one of the bulbs drawn is defective. Which of the following is the correct result?

a) \[ \frac{\text{choose}(3,1) \times \text{choose}(12,3)}{\text{choose}(12,4)} \]

b) \[ \frac{\text{choose}(3,1) \times \text{choose}(12,3)}{\text{choose}(15,4)} \]

c) \[ \frac{\text{choose}(3,1)}{\text{choose}(15,4)} \]

d) \[ \frac{3!}{12!} \]
Example 4

Suppose a box contains 3 defective light bulbs and 12 good bulbs. Suppose we draw a simple random sample of 4 light bulbs,

1. What is the probability that none of bulbs drawn are defective?

2. What is the probability that at least one of the bulbs drawn is defective?
Example 5

Suppose we select randomly 4 marbles drawn from a bag containing 8 white and 6 black marbles.

1. What is the probability that half of the marbles drawn are white?

2. What is the probability that at least 2 of the marbles drawn are white?