MATH 3339 - 03 15951
Statistics for the Sciences
Introduction and Probability; 3.4 - 3.6

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Lecture 5 - 3339
Outline

1. Conditional Probability
2. Bayes’ Rule
3. Examples
Conditional Probability

Let $A$ and $B$ be events with $P(B) > 0$. The **conditional probability** of $A$, given $B$ is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**General rule for multiplication**: For any two events $E$ and $F$, $P(E \cap F) = P(E) \times P(F|E)$ or $P(E \cap F) = P(F) \times P(E|F)$. 

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1. When do I add and when do I multiply?
   - Add when finding the chance of events A or B or both happening.
     \[ P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
   - Multiply when finding the chance that both events A and B happen.
     \[ P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B, \text{ given } A) = P(A)P(B|A) \]
Two Frequently Asked Questions

2. What’s the difference between disjoint (mutually exclusive) and independent?
   ▶ Two events are disjoint if the occurrence of one prevents the other from happening.
     \[ P(A \cap B) = 0 \]
   ▶ Two events are independent if the occurrence of one does not change the \textit{probability} of the other.
     \[ P(A|B) = P(A) \]
Example

Thirty percent of the students at a local high school face a disciplinary action of some kind before they graduate. Of those "felony" students, 45% go on to college. Of the ones who do not face disciplinary action 60% go on to college.

1. Show if events {faced disciplinary action} and {went to college} are independent or not.
Dogs and Cats

The probability of owning a dog is 0.6, the probability of owning a cat is 0.4. The probability of owning a dog and a cat is 0.24.

1. What is the probability that out of cat owners, they also own a dog?

2. What is the probability that out of dog owners, they also own a cat?

3. Are "owning a dog" and "owning a cat" independent events?
Buyers of Computers

Approximately 5 months after the introduction of the iMac, Apple reported that 32% of iMac buyers were first-time computer buyers. At the same time, approximately 5% of all computer sales were of iMacs. Of buyers who did not purchase an iMac, approximately 40% were first-time computer buyers. Let \( A \) = the event bought an iMac and \( B \) = the event of first-time computer buyer

1. What is the probability of a person buying an iMac, \( P(A) \)?

2. What is the probability that a person is a first-time computer buyer, given they bought an iMac, \( P(B|A) \)?

3. What is the probability that a person bought an iMac and is a first-time computer buyer, \( P(A \cap B) \)?
4. What is the probability of a person buying an iMac, given they are first-time buyers?
The probability of a person buying an iMac, given they are first-time buyers is an example of using **Bayes’ rule**.

Given a prior (initial) probability then from sources we obtain additional information about the events.

From these events we revise the probabilities and get a posterior probability.

This is an application of the General Multiplication Rule.

It might be easier to use either the tree diagram to calculate this probability.
Bayes’ Rule

Let $A$ and $B_1, B_2, \ldots, B_k$ be pairwise disjoint events such that each $P(B_i) > 0$ and $\Omega = B_1 \cup B_2 \cup \ldots \cup B_k$ and assume $P(A) > 0$. Then for each $i$,

$$P(B_i | A) = \frac{P(A | B_i) P(B_i)}{P(A)}$$
Aircraft Disappearance

Seventy percent of the light aircraft disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared.

a) If it has an emergency locator, what is the probability that it will not be discovered?

b) If it does not have an emergency locator, what is the probability that it will be discovered?
A clothing store targets young customers (ages 18 through 22) wishes to determine whether the size of the purchases related to the method payment. Suppose a customer is picked at random. The following is 300 customers the amount of the purchase and method payment.

<table>
<thead>
<tr>
<th></th>
<th>Cash</th>
<th>Credit</th>
<th>Layaway</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under $40</td>
<td>60</td>
<td>30</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>$40 or more</td>
<td>40</td>
<td>100</td>
<td>60</td>
<td>200</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>130</td>
<td>70</td>
<td>300</td>
</tr>
</tbody>
</table>
Example

1. What is the probability that the customer paid with a credit card?

2. What is the probability that the customer purchased under $40?

3. What is the probability that the customer paid with credit card given that the purchase was under $40?

4. What is the probability that the customer paid with credit card and that the purchase was under $40?

5. Are type of payment and amount of purchase independent?