Outline

1. Estimating Variance and Standard Deviation
2. Hypothesis Tests
3. Hypotheses
4. Decision of the Test
5. Errors
Popper Set Up

- Fill in all of the proper bubbles.
- Make sure your ID number is correct.
- Make sure the filled in circles are very dark.
- This is popper number 26.
A sample size of $n$ is drawn from a Normal population with variance $\sigma^2$.

The sample variance $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$. We can obtain a value of $s^2$ from the sample.

This computed sample variance is used as a point estimate of $\sigma^2$. Hence the statistic $s^2$ is an estimator of $\sigma^2$. 
The Distribution for Variance

\[ x^2 = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1} \]  See section 6.7 or the extra notes on CASA.

- Where \( X^2 \) has the chi-square distribution with degrees of freedom \( \nu = n - 1 \).

- Find \( P(X^2 \leq 15.033) \) with \( \nu = 6 \) using chi-square table and R.

- Find \( c \) such that \( P(X^2 \geq c) = 0.02 \) with \( \nu = 10 \).
Setting up the Confidence Interval

Thus: \[ P\left( \chi^2_{1-\alpha/2,n-1} < \frac{(n-1)S^2}{\sigma^2} < \chi^2_{\alpha/2,n-1} \right) = 1 - \alpha \]

Similarly to the confidence intervals of \( \mu \) and \( p \) we want to solve for \( \sigma^2 \).
Confidence Interval for $\sigma^2$

A 100$(1 - \alpha)\%$ confidence interval for the variance $\sigma^2$ of a normal population has lower limit

$$\frac{(n - 1)s^2}{\chi^2_{\alpha/2, n-1}}$$

and upper limit

$$\frac{(n - 1)s^2}{\chi^2_{1-\alpha/2, n-1}}$$

A confidence interval for $\sigma$ has lower and upper limits that are the square roots of the corresponding limits in the interval for $\sigma^2$, where $\alpha/2$ is the area in the upper tail of the chi-square distribution.
A coffee machine dispenses coffee into paper cups. Here are the amounts measured in a random sample of 20 cups.

9.9, 9.7, 10.0, 10.1, 9.9, 9.6, 9.8, 9.8, 10.0, 9.5, 9.7, 10.1, 9.9, 9.6, 10.2, 9.8, 10.0, 9.9, 9.5, 9.9

- Determine a 95% confidence interval for the mean amount of coffee dispensed from this machine.
- Determine a 95% confidence interval for the standard deviation of the amount of coffee dispensed from this machine.
> lcl = (length(coffee) - 1) * var(coffee) / qchisq(0.025, (length(coffee) - 1), lower.tail = F)
> ucl = (length(coffee) - 1) * var(coffee) / qchisq(0.025, (length(coffee) - 1))
> c(lcl, ucl)
[1] 0.02281421 0.08415187
> sqrt(c(lcl, ucl))
[1] 0.1510437 0.2900894
Example College Placement Tests

A random sample of 20 students obtained a mean of $\bar{x} = 72$ and a variance of $s^2 = 16$ on a college placement test in mathematics. Assuming the scores to be normally distributed, construct a 98% confidence interval for the variance.
Chapter 8: Hypothesis test
We believe the mean body temperature to be 98.6°F. But is the true population mean body temperature really 98.6°F?

University of Maryland researchers obtained temperatures from 100 healthy adults.

The following is the characteristics of the sample:
- The distribution is approximately bell shaped.
- The sample mean is 98.2°F.
- The standard deviation is $\sigma = 0.62°F$.
- The sample size is $n = 100$. 
What is the actual mean body temperature?

- It is commonly believed that the mean body temperature is 98.6°F, but the researchers in Maryland suggest that it might be less.

- Could the mean body temperature actually be less than 98.6°F?

- Could the sample mean of $\bar{x} = 98.2°F$ be a result of a chance sample fluctuation?

- What is the sampling distribution of $\bar{X}$?
Hypothesis Test

- To assess the evidence provided by data about some claim concerning a population.

- The reasoning is based on what would happen if we repeated the sample or experiment many times.

- The test of significance answers the question: "Is the observed effect due to chance?"
Components of a significance test

- Null and alternative hypothesis
- Rejection region
- Test Statistic
- P-value
- Decision of test
- Conclusion in context of the test
Null Hypothesis of significant tests

- The null hypothesis is the statement that is assumed to be true. We assume “no effect” or “no difference” for the parameter tested.

- Abbreviate the null hypothesis by $H_0$.

- From mean body temperature example, $H_0 : \mu = 98.6^\circ F$.

- For a significant test of the mean, the null hypothesis is always equal to some value of what we assume the mean to be.

- The null hypothesis is always $H_0 : \mu = \mu_0$, where $\mu_0$ is some value that is assumed to be the true mean.
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The alternative hypothesis is the statement we hope or suspect is true instead of the null hypothesis.

Abbreviate the alternative hypothesis by $H_a$.

From the mean body temperature example, $H_a : \mu < 98.6°F$.

The test of significance is designed to assess the strength of the evidence against the null hypothesis.
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Possible values for the Alternative Hypothesis

There are three possible ways that we would want to test against the null hypothesis.

1. Test to prove that the mean is really lower than what is assumed. This is called a **left-tailed test**. \( H_a : \mu < \mu_0 \)

2. Test to prove that the mean is greater than what is assumed. This is called a **right-tailed test**. \( H_a : \mu > \mu_0 \)

3. Test to prove that the mean is not equal (either higher or lower) than what is assumed. This is called a **two-tailed test**. \( H_a : \mu \neq \mu_0 \)
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Popper 26 Questions

1. Last year, a company’s service technicians took an average of 2.6 hours to respond to trouble calls from business customers who had purchased service contracts. Do this year’s data show a different average response time?
   a) $H_0 : \mu = 2.6$ and $H_a : \mu \neq 2.6$
   b) $H_0 : \mu = 2.6$ and $H_a : \mu > 2.6$
   c) $H_0 : \mu = 2.6$ and $H_a : \mu < 2.6$
   d) $H_0 : \mu = 2.6$ and $H_a : \mu = 0$

2. The manager of an automobile dealership is considering a new bonus plan designed to increase sales volume. Currently, the mean sales volume is 14 automobiles per month. The manager wants to conduct a research study to see whether the new bonus plan increases sales volume. To collect data on the plan, a sample of sales personnel will be allowed to sell under the new bonus plan for a one-month period.
   a) $H_0 : \mu = 14$ and $H_a : \mu \neq 14$
   b) $H_0 : \mu = 14$ and $H_a : \mu > 14$
   c) $H_0 : \mu = 14$ and $H_a : \mu < 14$
   d) $H_0 : \mu = 14$ and $H_a : \mu = 0$
Since there are only two hypotheses, there are only two possible decisions.

- **Reject** the null hypothesis in favor of the alternative hypothesis. (RH0)

- **Fail to** reject the null hypothesis. (FTRH0)

- We will **never** say that we accept the null hypothesis.
Suppose a person is on trial for murder. In the U.S. court of law what is the assumption?

- Assumption - Innocent (Null hypothesis)
- Need to prove - Guilty (Alternative hypothesis)
- Jury’s choices for a decision - Guilty or not guilty (Conclusion)
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## Analogy of the decision

Can the jury make a wrong decision?

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<th>Jury’s Decision</th>
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Analogy of the decision

- According to the person on trial which error would be worse to get?
  - This error is called the type I error: Rejecting the null hypothesis when in fact it is true.
  - Since this is the worst conclusion we try to control for this error.
  - $P(\text{Type I error}) = \alpha$ the level of significance.
  - By predetermining $\alpha$ usually 0.05, we are saying that we make this type I error only 5% of the time.
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Errors in our decision

In the same way we can make an error in our decisions.

<table>
<thead>
<tr>
<th>Our Decision</th>
<th>Correct Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject $H_0$</td>
<td>Type I Error</td>
</tr>
<tr>
<td>Fail to reject $H_0$</td>
<td>Correct</td>
</tr>
</tbody>
</table>

Thus by determining $\alpha$, the level of significance, we try to control for the Type I error.
Type I and Type II Errors

- If we reject $H_0$ when it is true this is a **Type I error**.

- If we do not reject $H_0$ when it is false, this is a **Type II error**.

In the example of the mean body temperature:

- **Type I error**: We would conclude that the mean body temperature is less than 98.6 degrees, when in fact it truly is 98.6.

- **Type II error**: We would conclude that the mean body temperature is not less than 98.6 degrees, when in fact the mean body temperature is less than 98.6.
You are considering whether or not to play the lottery with your favorite numbers. What situations denote a type I and II errors?

<table>
<thead>
<tr>
<th></th>
<th>$H_0$ is true (Won’t win)</th>
<th>$H_a$ is true (Would win)</th>
</tr>
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<tbody>
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<td>Reject $H_0$ (buy a ticket)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fail to reject $H_0$ (don’t buy)</td>
<td></td>
<td></td>
</tr>
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</table>
A can of Pepsi is supposed to have a mean volume of 12 ounces. Both overfilling and under-filling are undesirable. If either occurs, the machine that fills the cans has to be readjusted.

3. The bottling company wants to set up a hypothesis test so that the machine has to be readjusted if the null hypothesis is rejected. Set up the null and alternative hypothesis for this test.

   a) $H_0 : \mu = 12$ and $H_a : \mu \neq 12$
   b) $H_0 : \mu = 12$ and $H_a : \mu < 12$
   c) $H_0 : \bar{x} = 12$ and $H_a : \bar{x} \neq 12$
   d) $H_0 : \mu = 12$ and $H_a : \mu > 12$

4. Once completing the test, they readjusted the machine but they did not have to. This is an example of:

   a) Type I error
   b) Type II error
   c) Type III error
   d) correct decision