MATH 2331 - 20947

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Lecture : MoWeFr 10:00AM-11:00AM in CBB 104
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Vector equations

Section 1.3
**Vector**

### Vectors in $\mathbb{R}^n$

Vectors with $n$ entries: $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$, a matrix with one column.

### Geometric description of $\mathbb{R}^2$

Vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is depicted as the arrow connecting the origin of the axes $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to the point $(x_1, x_2)$ in the plane.

$\mathbb{R}^2$ is the set of all points in the plane.
Operations with vectors

**Sum**

Given vectors \( u, v \in \mathbb{R}^n \), vector \( u + v \in \mathbb{R}^n \) is:

\[
\begin{bmatrix}
u_1 \\
u_2 \\
\vdots \\
u_n
\end{bmatrix} +
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_n
\end{bmatrix} =
\begin{bmatrix}
u_1 + v_1 \\
u_2 + v_2 \\
\vdots \\
u_n + v_n
\end{bmatrix}
\]

**Multiplication by scalar**

Given vectors \( u \in \mathbb{R}^n \), and scalar \( c \in \mathbb{R} \) is:

\[
cu = c
\begin{bmatrix}
u_1 \\
u_2 \\
\vdots \\
u_n
\end{bmatrix} =
\begin{bmatrix}
cu_1 \\
cu_2 \\
\vdots \\
cu_n
\end{bmatrix}
\]
Parallelogram rule

If \( \mathbf{u} \) and \( \mathbf{v} \) in \( \mathbb{R}^2 \) are represented as arrows to points in the plane, then \( \mathbf{u} + \mathbf{v} \) corresponds to the diagonal of the parallelogram with \( \mathbf{u} \) and \( \mathbf{v} \) as two sides.

Example: Let \( \mathbf{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \) and \( \mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \). \( \mathbf{u} + \mathbf{v} \) is:
Example

Let \( \mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \). Draw \( \mathbf{u} \), \( 2\mathbf{u} \), and \( \frac{-3}{2}\mathbf{u} \).
Linear combinations of vectors

Given vectors $v_1, v_2, \ldots, v_p$ in $\mathbb{R}^n$ and given scalars $c_1, c_2, \ldots, c_p$ in $\mathbb{R}$, the vector $y$ defined by

$$y = c_1 v_1 + c_2 v_2 + \cdots + c_p v_p$$

is called a **linear combination** of $v_1, v_2, \ldots, v_p$ using weights $c_1, c_2, \ldots, c_p$. 
Example

Let \( \mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \), \( \mathbf{a}_2 = \begin{bmatrix} 4 \\ 2 \\ 14 \end{bmatrix} \), \( \mathbf{a}_3 = \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} \), and \( \mathbf{b} = \begin{bmatrix} -1 \\ 8 \\ -5 \end{bmatrix} \).

Determine if \( \mathbf{b} \) is a linear combination of \( \mathbf{a}_1 \), \( \mathbf{a}_2 \), and \( \mathbf{a}_3 \).
Review of the example

\( a_1, a_2, a_3 \) and \( b \) are columns of the augmented matrix

\[
\begin{bmatrix}
1 & 4 & 3 & -1 \\
0 & 2 & 6 & 8 \\
3 & 14 & 10 & -5 \\
\end{bmatrix}
\]

Solution to

\[ x_1 a_1 + x_2 a_2 + x_3 a_3 = b \]

is found by solving the linear system whose augmented matrix is

\[
\begin{bmatrix}
a_1 & a_2 & a_3 & b \\
\end{bmatrix}
\]
Linear combinations and vector equation

**Vector Equation**

A vector equation

\[ x_1a_1 + x_2a_2 + \cdots + x_na_n = b \]

has the same solution set as the linear system whose augmented matrix is

\[
\begin{bmatrix}
a_1 & a_2 & \cdots & a_n & b \\
\end{bmatrix}
\]

In particular, \( b \) is a linear combination of \( a_1, a_2, \ldots, a_n \) if and only if there is a solution to the linear system corresponding to the augmented matrix.