If you miss a homework, you will receive a zero, but you will get full amount of points for the two lowest homework grades at the end of the course.

The lower test grade will be replaced by the grade of the final exam (x100/150), if the latter is higher.
More on matrix-vector product

Theorem

If $A$ is an $m \times n$ matrix, $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^n$, and $c$ is a scalar, then:

1. $A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$;
2. $A(c\mathbf{u}) = cA\mathbf{u}$. 

Solutions sets of linear systems

Section 1.5
Definition

Let $A$ be a $m \times n$ matrix. System $Ax = 0$, where $0$ is the zero vector in $\mathbb{R}^m$, is called **homogeneous system**.

**Example:**

\[
\begin{align*}
    x_1 &+ 10x_2 &= 0 \\
    2x_1 &+ 20x_2 &= 0 
\end{align*}
\]

The homogeneous system $Ax = 0$ always has the **trivial solution**: $x = 0$. 
Homogeneous System: Nontrivial Solutions

**Definition**

Nonzero vector solutions are called **nontrivial solutions**.

Does the system in the previous slide have **nontrivial** solutions?

A homogeneous equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions if and only if the system of equations has infinitely many solutions, i.e. if the system has at least one free variable.
Example

Determine if the following homogeneous system has nontrivial solutions and then describe the solution set:

\[
\begin{align*}
2x_1 &+ 4x_2 - 6x_3 = 0 \\
4x_1 &+ 8x_2 - 10x_3 = 0
\end{align*}
\]
Example (cont.)

\[
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix}
= \begin{bmatrix}
    -2x_2 \\
    x_2 \\
    0
\end{bmatrix}
= x_2 \begin{bmatrix}
    -2 \\
    1 \\
    0
\end{bmatrix}
= x_2 \mathbf{v}
\]

Graphical representation:

solution set = \text{span}\{\mathbf{v}\} = \text{line through 0 in } \mathbb{R}^3
Nonhomogeneous System

Definition
Let \( A \) be a \( m \times n \) matrix. System \( Ax = b \), where \( b \neq 0 \), is called nonhomogeneous system.

Example: Describe the solution set of

\[
\begin{align*}
2x_1 + 4x_2 - 6x_3 &= 0 \\
4x_1 + 8x_2 - 10x_3 &= 4
\end{align*}
\]
\[
x = \begin{bmatrix} 6 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = p + x_2v
\]

Graphical representation: line parallel to \( v \) passing through the tip of vector \( p \).
Recap of the previous examples

- Solution of $Ax = 0$:
  \[
  x = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = x_2v
  \]

  $x = x_2v$ = parametric equation of line passing through $0$ and $v$

- Solution of $Ax = b$:
  \[
  x = \begin{bmatrix} 6 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = p + x_2v
  \]

  $x = p + x_2v$ = parametric equation of line passing through the tip of $p$ and parallel to $v$

The solution sets of $Ax = 0$ and $Ax = b$ are parallel.
Nonhomogeneous System: Theorem

Theorem

Suppose the equation \( Ax = b \) is consistent for some given \( b \), and let \( p \) be a solution. Then the solution set of \( Ax = b \) is the set of all vectors of the form \( w = p + v_h \), where \( v_h \) is any solution of the homogeneous equation \( Ax = 0 \).

Example: Describe the solution set of \( 2x_1 - 4x_2 - 4x_3 = 0 \); compare it to the solution set \( 2x_1 - 4x_2 - 4x_3 = 6 \).
Recap of the previous example

- Solution of the homogeneous system:

\[
x = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = x_2 u + x_3 v
\]

\( x_2 u + x_3 v \) is the equation of a plane passing through \( \mathbf{0} \) and containing \( u, v \)

- Solution of the nonhomogeneous system:

\[
x = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = p + x_2 u + x_3 v
\]

\( p + x_2 u + x_3 v \) is the equation of a plane passing through the tip of \( p \) and parallel to \( u \) and \( v \)
Example (cont.)

Graphical representation:

The solution sets of $Ax = 0$ and $Ax = b$ are parallel.