MATH 2331 - 17571

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Lecture : TuTh 1:00PM-2:30PM in SEC 203
Office hours : TuTh 2.30PM-4PM and BY APPOINTMENT

- Use your UH e-mail account only!
- Exam 1 will be on Thursday 09/27 IN CLASS (75 min.).
- Exam 1 will be on Sec. 1.1-1.9.
- Review for the exam on Tuesday 02/25.
Introduction to Linear Transformations

Section 1.8
A transformation $T$ from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a rule that assigns to each vector $x$ in $\mathbb{R}^n$ a vector $T(x)$ in $\mathbb{R}^m$. 

$T : \mathbb{R}^n \rightarrow \mathbb{R}^m$

transformation  
“machine”
Transformations have many applications - including computer graphics.
Consider transformation:

\[ T : \mathbb{R}^n \rightarrow \mathbb{R}^m \]
\[ T : x \mapsto T(x) \]

\( \mathbb{R}^n \) is the **domain** of \( T \).

\( \mathbb{R}^m \) is the **codomain** of \( T \).

\( T(x) \) in \( \mathbb{R}^m \) is the **image** of \( x \) under the transformation \( T \).

Set of all images \( T(x) \) is the **range** of \( T \).
Define $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ such that

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right) = \begin{bmatrix} 3 + x_1^2 - x_3 \\ -x_3x_2 \end{bmatrix}$$

Given $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 5 \end{bmatrix}$, find $T(\mathbf{x})$. 
Example

Define $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ by

$$T(x) = x_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Given $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, find $T(x)$. 
Example (cont.)
Another Way to View $A\mathbf{x} = \mathbf{b}$

Matrix $A$ is an object acting on $\mathbf{x}$ to produce a new vector $\mathbf{b}$.

Matrix Transformations

Suppose $A$ is $m \times n$. Solving $A\mathbf{x} = \mathbf{b}$ amounts to finding all $\mathbf{x}$ in $\mathbb{R}^n$ which are transformed into vector $\mathbf{b}$ in $\mathbb{R}^m$ through multiplication by $A$.

multiply by $A$
Example

Given:

\[ A = \begin{bmatrix} 1 & -2 & 3 \\ -5 & 10 & -15 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -10 \end{bmatrix}, \quad c = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \]

define a matrix transformation \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \) by \( T(x) = Ax \).

a. Find an \( x \) in \( \mathbb{R}^3 \) whose image under \( T \) is \( b \).
Given:

\[
A = \begin{bmatrix}
1 & -2 & 3 \\
-5 & 10 & -15
\end{bmatrix}, \quad b = \begin{bmatrix}
2 \\
-10
\end{bmatrix}, \quad c = \begin{bmatrix}
3 \\
0
\end{bmatrix},
\]

define a transformation \( T : \mathbb{R}^3 \to \mathbb{R}^2 \) by \( T(x) = Ax \).

b. Is there more than one \( x \) under \( T \) whose image is \( b \)?

(*uniqueness problem*)
Example (cont.)

Given:

\[
A = \begin{bmatrix}
1 & -2 & 3 \\
-5 & 10 & -15
\end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -10 \end{bmatrix}, \quad c = \begin{bmatrix} 3 \\ 0 \end{bmatrix},
\]

define a transformation \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \) by \( T(x) = Ax \).

c. Determine if \( c \) is in the range of the transformation \( T \). *(existence problem)*