MATH 2331 - 20947

Vladimir Yushutin

yushutin@math.uh.edu

Office : PGH 606

Lecture : MoWeFr 10:00AM-11:00AM in CBB 104

Office hours : Fr 3:00PM-4:00PM and BY APPOINTMENT
Let $T : V \rightarrow W$ be a linear transformation.

**Definition**

The *kernel* of $T$ is the set of all vectors $u$ in $V$ such that $T(u) = 0$.

So if $T(x) = Ax$, $\text{Nul } A = \text{kernel of } T$.

**Definition**

The *range* of $T$ is the set of all vectors in $W$ of the form $T(u)$ where $u$ is in $V$.

So if $T(x) = Ax$, $\text{Col } A = \text{range of } T$. 
Rank

Section 4.6
Row Space

The **row space** of an $m \times n$ matrix $A$, denoted by Row $A$, is the set of all linear combinations of the rows of $A$.

$$\text{Row } A = \text{Col } A^T$$

**Theorem**

*The row space of an $m \times n$ matrix $A$ is a subspace of $\mathbb{R}^n$.*

Note that we can also consider the Nul $A^T$ finding a symmetry in positions of $A$ and $A^T$. 
Definition

*Rank* of a linear transformation is the number of pivots in the standard matrix.

Theorem (4 subspaces for $T$)

*For a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ with a standard matrix $A$:

$$\dim \text{Row } A = r \quad \quad \quad \dim \text{Col } A = r$$

$$\dim \text{Nul } A = n - r \quad \quad \quad \dim \text{Nul } A^T = m - r$$

where $r = \text{rank } A$ is the rank of $T$.

Fundamental subspaces can intersect by the zero vector only.
Bases for the 4 subspaces of $T$

1. Find a standard matrix for $T$: $[T(e_1), ..., T(e_n)] = A$
2. Row reduce $A$ to the echelon form $B$.
3. Pivot columns of $A$(not $B$!) form a basis for the Col $A$. Corresponding pivot rows of $A$ form a basis for the Row $A$.
4. Vectors from the parametric form of the solution set of $Bx = 0$ constitute a basis for Nul $A$.
5. To find a basis for Nul $A^T$ row reduce $A^T$ and find the solution to $A^T x = 0$. Vectors from the parametric form constitute a basis for Nul $A^T$. 